Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A ship, of mass 5000 tonnes, is moving through the sea at a constant speed of 15 km h^{-1}.

   (a) Calculate the momentum of the ship, in the form $a \times 10^n$, where $0 \le a < 10$ and n is an integer. State the units of your answer. **(3 marks)**

   Given that there is a constant force of magnitude 4000 N acting against the ship due to air and water resistances,

   (b) find the rate, in kW, at which the ship's engines are working. **(3 marks)**

2. Two small smooth spheres P and Q are moving along a straight line in opposite directions, with equal speeds, and collide directly. Immediately after the impact, the direction of P's motion has been reversed and its speed has been halved. The coefficient of restitution between P and Q is e.

   (a) Express the speed of Q after the impact in the form $au(be + c)$, where a, b and c are constants to be found. **(4 marks)**

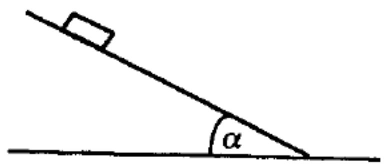
   (b) Deduce the range of values of e for which the direction of motion of Q remains unaltered. **(3 marks)**

3. **i** and **j** are perpendicular unit vectors in a horizontal plane. At a certain instant, a particle P of mass 1·8 kg is moving with velocity $(24\mathbf{i} - 7\mathbf{j})$ ms^{-1}.

   (a) Calculate the kinetic energy of P at this instant. **(3 marks)**

   P is now subjected to a constant retardation. After 10 seconds, the velocity of P is $(-12\mathbf{i} + 3{\cdot}5\mathbf{j})$ ms^{-1}.

   (b) Calculate the work done by the retarding force over the 10 seconds. **(5 marks)**

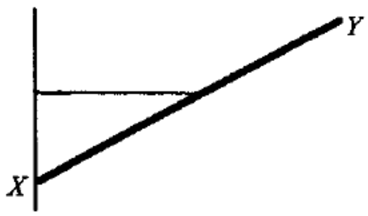
4. 

   A small block of wood, of mass 0·5 kg, slides down a line of greatest slope of a smooth plane inclined at an angle α to the horizontal, where $\sin \alpha = \frac{2}{5}$. The block is given an initial impulse of magnitude 2 Ns, and reaches the bottom of the plane with kinetic energy 19 J.

   (a) Find, in J, the change in the potential energy of the block as it moves down the plane. **(3 marks)**

   (b) Hence find the distance travelled by the block down the plane. **(4 marks)**

   (c) State two modelling assumptions that you have made. **(2 marks)**

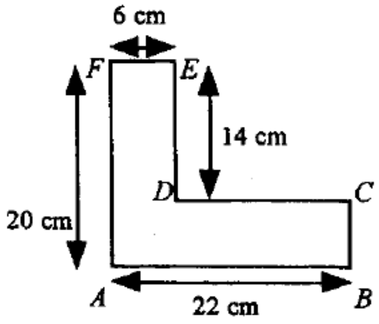
5. 

A uniform rod XY, of length $2a$ and mass m, is connected to a vertical wall by a smooth hinge at the end X. A horizontal light inelastic string connects the mid-point of XY to the wall and the rod is in equilibrium in this position.

(a) Draw a diagram to show all the forces acting on the rod. **(4 marks)**

Given that the tension in the horizontal string is of magnitude $2mg$,

(b) find the angle which XY makes with the vertical. **(5 marks)**

6. 

The diagram shows a uniform lamina $ABCDEF$.

(a) Calculate the distance of the centre of mass of the lamina from (i) AF, (ii) AB. **(6 marks)**

The lamina is hung over a smooth peg at D and rests in equilibrium in a vertical plane.

(b) Find the angle between CD and the vertical. **(4 marks)**

7. A particle P moves in a straight line so that its displacement s metres from a fixed point O at time t seconds is given by the formula $s = t^3 - 7t^2 + 8t$.

(a) Find the values of t when the velocity of P equals zero, and briefly describe what is happening to P at these times. **(5 marks)**

(b) Find the distance travelled by P between the times $t = 3$ and $t = 5$. **(3 marks)**

(c) Find the value of t when the acceleration of P is -2 ms^{-2}. Briefly explain the significance of a negative acceleration at this time. **(3 marks)**

8. A particle P is projected from a point O with initial velocity $(3{\cdot}5\mathbf{i} + 12\mathbf{j})$ ms^{-1} and moves under gravity. **i** and **j** are unit vectors in the horizontal and vertical directions respectively.

(a) Find the initial speed of P. **(2 marks)**

(b) Show that the position vector **r** m of P at time t seconds after projection is given by

$$\mathbf{r} = 3{\cdot}5t\,\mathbf{i} + (12t - 4{\cdot}9t^2)\mathbf{j}.$$ **(4 marks)**

(c) Find the horizontal distance of P from O at each of the times when it is 4·4 m vertically above the level of O. **(5 marks)**

In a refined model of the motion of P, the position vector of P at time t seconds is taken to be

$$\mathbf{r} = 3{\cdot}5t\,\mathbf{i} + (12t - t^3)\mathbf{j} \text{ m}.$$

(d) Using this model, find the position vector of the highest point reached by P. **(4 marks)**